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Summary:

Topology optimization is a very powerful tool to develop new concepts and has been widely used in engineering problems involving static loading conditions. However, there has been relatively little work for topology optimization of industrial size non-linear dynamic systems. The main issues are non-linear interactions among the material properties, contacts between parts, large strain-rates, transient behavior, etc. A hybrid cellular automata based method, combining cellular automata theory with the fully loaded design concept, has been demonstrated to be effective in generating new concept designs. This method is called hybrid cellular automata (HCA) and is used as the core algorithm to optimize the topology. This method is implemented in the LS-DYNA framework and would be available shortly. The method has shown encouraging results while solving many engineering problems. In this paper, the details of the methodology and a few engineering examples are provided to demonstrate some capabilities of the code. The main problem solved using the proposed methodology is the development of an optimal topology for a 1 million element box-shaped design domain subject to impact.

Keywords:

Topology Optimization, Non-linear, Crashworthiness, Hybrid cellular automata, Dynamic problems
1 Introduction

Topology optimization is a first-principle based optimization method to develop new concepts in engineering problems. Topology optimization in structures has been studied since the 1970s resulting in many books and numerous papers. The books by Rozvany [1] and Bendsøe and Sigmund [2] provide a very comprehensive and contemporary survey of optimization techniques used in topology optimization. Most previous studies [3, 4] in topology optimization have focused on designing structures with static loading conditions but there is relatively little work on handling problems involving dynamic loads, like those observed in crashworthiness optimization [5]. The topology optimization in the context of crashworthiness is a very complex problem due to non-linear interactions among material non-linearities, geometry, and transient nature of boundary conditions.

The most efficient topology optimization methods use sensitivity information (optimality criterion based methods, Rozvany [1], Bendsøe and Kikuchi [6]) to drive the search for an optimum. Sensitivity calculations are computationally inexpensive for linear-static problems but not for the problems that involve non-linearities. To use the same set of topology optimization methods, one needs to explicitly calculate sensitivities which is practically infeasible due to very high computational cost involved with simulations. Thus the theory used to solve the linear-static load cases, though quite mature, is not practical for the crashworthiness problems and alternate methods need to be explored.

Previously different approaches have been adopted by authors to solve topology optimization with nonlinearities. Pedersen used the Method of Moving Asymptotes for crashworthiness optimization of two-dimension structures [7]. They used a quasi-static nonlinear FEA to account for geometric nonlinearities to handle large deformation and rotation of plastic beam elements. However, the method ignored the contact between elements arising due to nonlinear behavior of the structures. Soto [8, 9] presented a heuristics based method using a prescribed plastic strain or stress criterion to vary the density to achieve the desired stress or strains with a constraint on mass. However, this method could not be generalized to solid structures. Pedersen [10] used beam elements to handle topology in crashworthiness optimization. Forsberg and Nilsson [11] proposed two algorithms to get a uniform distribution of the internal energy density in the structure. In the first method, they deleted inefficient elements and in the second method they updated the thicknesses of the shell elements. This method also was limited to a small set of optimization problems. Shin et al. [12] proposed an equivalent static load method where they calculated an equivalent static load for the dynamic problem and then used the linear-static topology optimization techniques to find the optimal topology. The main difficulty in this method is the requirement to accurately compute the equivalent loads.

A heuristic topology optimization method developed at the University of Notre Dame, known as hybrid cellular automata [13], has shown the most potential in handling topology optimization problem for crashworthiness problems. This method updates the density of elements based on the information from its neighbors. No gradient information was required. The simplicity and effectiveness of this method for both two- and three-dimensional problems made it the first choice for implementation in LS-OPT® [14] for topology optimization.

2 Hybrid Cellular Automata

According to Mathworld [15] “A cellular automaton (CA) is a collection of ‘coloured’ cells on a grid of specified shape that evolves through a number of discrete time steps according to a set of rules based on the states of neighbouring cells. The rules are then applied iteratively for as many time steps as desired”. In the conventional cellular automata, the state of a cell is determined based on the local information. However for the structural optimization problems, the state of a cellular automaton is defined by rules that combine the local neighborhood information with the field variables that are calculated globally e.g., stress, internal energy density, etc. Thus the state of the cell is determined using a combination of local and global information and hence the name hybrid cellular automata [13].

2.1 Components of a HCA

While any arbitrary shape can be used for the cells, mostly the shape of the cells is kept same as the shape of elements. The state of a cell defines its characteristics and comprises of both design and field variables (e.g. stress). Since the search is driven by the rules defined on the neighbours of each cell, it is very important to pick the neighbours appropriately. A few common strategies to pick
neighbours are shown in Figure 2-1. The choice of neighbourhood definition must be homogenous throughout the design domain.

The rules correlate the state of a CA lattice with the states of its neighbours at any time step. An example of a rule is that the internal energy density \(U_i\) of the \(i\)th CA lattice is the average of its von Neumann neighbourhood with \(n\) neighbours,

\[
U_i = \frac{\sum_{j=1}^{n} U_j}{\sum_{j=1}^{n} 1}. \tag{1}
\]

In the context of HCA, the field variables are subjected to the neighbourhood update rule first and then the design variables are updated. This process is iterated until equilibrium is established among all state variables.

2.2 Design Problem

The typical goal of topology optimization is to obtain maximum utility of the material. Compliance and the strain energy density are the most commonly used objectives for linear-static problems. For dynamic problems like crashworthiness simulations, the structure needs to absorb maximum energy while maintaining the structural integrity and keeping the peak loads transmitted to the occupants low. Following the formulation proposed by Patel [16], the goal of obtaining uniform internal energy density in the structure is defined as the objective for optimization and the mass is constrained. This concept is similar to the fully-stressed design and uniform strain energy density approaches (Haftka and Gurdal [17], Patnaik and Hopkins [18]) that are well established in literature for linear-static problems. The optimization problem is formulated as,

\[
\min_x \sum_{i=1}^{N} \sum_{j=1}^{L} \left( w_j U_j(x_i) - U^* \right) \tag{2}
\]

subject to: \[ \sum_{i=1}^{N} \rho(x_i) V_i \leq M^* \tag{3} \]

\[ \rho_{\text{min}} \leq x_i \leq 1.0. \]

where \(U_i\) represents the internal energy density of the \(i\)th element, \(V_i\) is the volume of \(i\)th element, \(U^*\) represents a internal energy density set point, and there are \(L\) load cases.

A density based approach is used to define the design variables \(x\) for each cell such that each cell has its own material model. The solid isotropic material with penalization (SIMP [19]) model is used to interpolate the material properties as follows,

\[
\rho(x) = x \rho_0, \tag{4}
\]

\[
E(x) = x^p E_0, \tag{5}
\]

\[
\sigma(x) = x^q \sigma_0, \tag{6}
\]
\[ E_h(x) = x^0 E_{h0}, \]  

The subscript ‘0’ refers to the base material properties. The design variable \( x \), also known as relative density, varies from 0 to 1 where ‘0’ indicates void and ‘1’ represents full material. A more detailed description of the material model parameterization, one should refer to Bendsøe and Sigmund [2], and Patel [16]. The elements with design variable value less than a user-defined minimum value are deleted to improve numerical stability.

### 2.3 Implementation in LS-OPT/Topology

Figure 2-2 depicts a flowchart of the implementation of the HCA technology in the LS-OPT/Topology code.

![Flowchart of hybrid cellular automata based topology optimization method.](image)

Firstly, a mapping between the finite element mesh and the CA lattice is defined and the neighbours of each cell are identified. The CA state is initialized to satisfy the mass constraint. LS-DYNA® [20] simulations are carried out to evaluate the updated topology and field variables are obtained. The search is driven by applying neighbourhood rules on the CA lattice and updating the design variables. Two convergence criteria, number of iterations and total change in the design variables are used to terminate the search. More details can be obtained elsewhere [21].

### 3 Examples

The capabilities and performance of the LS-OPT/Topology code are demonstrated with the help of three examples.

#### 3.1 Statically Loaded Structure

The geometry, mesh, and the loading conditions are shown in Figure 3-1.

![Geometry and load condition for linear-static example](image)

While the bottom section is fixed, a uniform distributed load is applied on the top. The geometry is meshed using 125k elements (1mm element size). Each linear-static analysis using LS-DYNA implicit solver takes approximately 75s on a 2.66GHz Intel Xeon CPU with 4GB memory.
3.2 Dynamically Loaded Structure

The geometry shown in Figure 3-1 is also analyzed by considering a dynamic load case when a pole hits the block with an initial velocity of 8.9m/s. The simulation is analyzed for 3ms. For this example, a refined mesh of 1e6 elements (0.5mm element size) was used. LS-DYNA MPP explicit solver takes approximately 90min on 32 processors of a SGI cluster.

3.3 Extruded Curved Beam

This example simulated crash between a bumper and a wall. The initial geometry of the bumper was modeled using a beam supported by fixtures. The wall was assigned an initial velocity of 5m/s. Only half section was simulated by taking advantage of the symmetry. The design problem was shown in Figure 3-2. There were approximately 26k elements. The crash event was simulated for 20ms. A single layer of elements that was in touch with the wall was forced to be there by defining that as non-design part. The cross-section of the bumper was evolved using the extrusion method. The symmetry face was defined as the extruded section with 214 elements. Each LS-DYNA explicit simulation took approximately 190s on 6 CPUs.

Figure 3-2 Curved beam solved in extrusion mode.

For all examples aluminum is used as the material and corresponding properties are furnished in Table 1 and corresponding stress-strain curve is shown in Figure 3-3.

Table 1: Material properties of Al.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>MAT_PIECEWISE_LINEAR_PLASTICITY</td>
</tr>
<tr>
<td>Density</td>
<td>2.7e-9 tonnes/mm³</td>
</tr>
<tr>
<td>Young’s Mod.</td>
<td>7e4N/mm²</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>180N/mm²</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Figure 3-3: Stress-strain curve for Al.

4 Results and Discussion

This section discusses the results obtained for the three examples.
4.1 Statically Loaded Structure

The convergence history of the optimization process was shown with the help of the total internal energy density (IED) in the structure and the total change in the topology in Figure 4-1. There was significant drop in the IED till 10 iterations and afterwards the changes were relatively small. On the other hand, the variations in the topology were largely monotonic till convergence at 28th iteration.

![Convergence history of topology optimization for the statically loaded structure.](image)

**Figure 4-1:** Convergence history of topology optimization for the statically loaded structure.

![Density contours at iterations 5, 10, 15 (row 1), 20, 25, and 28 (row 2) of topology optimization when a fixed load is applied.](image)

**Figure 4-2:** Density contours at iterations 5, 10, 15 (row 1), 20, 25, and 28 (row 2) of topology optimization when a fixed load is applied.

![IED and von-Mises stress at iterations 5, 10, 15 (row 1), 20, 25 and 28 (row 2) for topology optimization of the statically loaded structure.](image)

**Figure 4-3:** IED and von-Mises stress at iterations 5, 10, 15 (row 1), 20, 25 and 28 (row 2) for topology optimization of the statically loaded structure.
The density contours depicting the evolution of the optimal topology were shown in Figure 4-2. The internal energy density and the von-mises stress fields were also plotted in Figure 4-3. As expected the topology optimization resulted in a columnar structure as the optimized shape. The material, IED, and the stress were also distributed quite homogeneously throughout the structure.

### 4.2 Dynamically Loaded Structure

The convergence history of the dynamically loaded structure was shown in Figure 4-4. There was largely monotonic change in the topology quantified by the sum of variables. Significant changes were observed initially but only small improvements in the topology were obtained after 35 iterations. On the other hand, the total internal energy density showed variations with iterations. The IED dropped rapidly initially but was relatively stabilized with some non-periodic oscillatory behavior.

![Figure 4-4: Convergence history of topology optimization of the dynamically loaded structure.](image)

The density and internal energy density distributions at different iterations were plotted in Figure 4-5.

![Figure 4-5: Density and IED contours at iterations10, 15, 20 (row 1), 30, 40 and 47 (row 2) when the structure is dynamically loaded.](image)

The final structure had a reasonably homogeneous distribution of density and IED. It was obvious that the structure was largely evolved by 30 iterations with relatively little re-distribution of material.
afterwards. More details about the final structure were shown in Figure 4-6 where only half and a quarter sections were visible. The double-arch formation inside the structure was clearly notable. This design was significantly different than the columnar structure obtained with static loads. This result was expected because the double-arch structure provides more stability with the dynamic loads by exploiting large deformations.

4.3 Extruded Curved Beam

The variation in the total internal energy density and the change in topology with the iteration were shown in Figure 4-7. The convergence took more iterations than the non-extrusion cases. The topology changed significantly initially and convergence trend was established only after 15 iterations. The smaller scale on the change in topology graph represented fewer cells in the lattice compared to those observed in non-extrusion examples. In terms of total internal energy density, the final structure performed comparatively to the initial structure as significant recovery of the IED was observed during the course of optimization.

![Figure 4-7: Convergence history of topology optimization for the extruded curved beam.](image)

The density fields at different iterations in 3-D structure and corresponding cross-sections that were designed were plotted in Figure 4-8. The final structure had an inverted U-shaped cross section. In the early phase, the beam had an I-shaped cross-section with low density material. The material from the interior was redistributed to the wall as evolution progressed. As was observed for other cases, the final structure was reasonably homogeneous in material distribution.

![Figure 4-8: Density contours in 3-D at iterations 10, 20 (row 1), 30, 40 (row 2), 50, and 60 (row 3) and density contours on the design cross-section in the topology optimization of the extruded geometry.](image)

5 Summary

A hybrid cellular automata based topology optimization method is implemented in the LS-DYNA framework and tested using a set of engineering examples. The code results in a reasonable homogeneous distribution of the material for all examples. The extrusion mode example requires more iterations (and simulations) for convergence though a reasonably well developed shape was evolved after 25 iterations for all examples.

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It is demonstrated that the LS-OPT/Topology code can easily and rapidly develop optimal topologies for industrial problems both for static and dynamic problems. The new implementation can also easily handle features like extruded sections, arbitrarily shaped design domains, and many parts in a model. While not demonstrated in this paper, the code is also capable of handling multiple load cases [21].

6 Literature